1. **(14 Marks) The following sorted data are effluent contaminant concentrations in mg/L:**

10	11	11	12	12
12	13	13	13	14
14	14	14	15	15
16	16	17	18	24

The sample mean is $\bar{x} = 14.200$ mg/L and the sample variance is $s^2 = 9.642$ (mg/L)^2.

1. **What is the median of these data?**

   A. 13
   B. 14
   C. 15
   D. 16
   E. 12

2. **What is the range of these data?**

   A. 13
   B. 14
   C. 15
   D. 16
   E. 12

3. **What is the coefficient of variation of these data? Give the answer in percent.**

   A. 18.50 percent
   B. 20.13 percent
   C. 21.87 percent
   D. 23.46 percent
   E. 25.71 percent

4. **What is the mode of these data?**

   A. 12
   B. 13
   C. 15
   D. 14
   E. 24

no	frequency
10	I
11	II
12	III
13	III
14	IIII
15	II
16	II
17	I
18	I
24	I

$s = 3.1$ $\bar{x} = 14.2$ median $= 14$ $\bar{x} >$ median right skewed

5. Using Pearson's skewness coefficient $Sk = \frac{3(\bar{x} - \text{median})}{s}$, what is the skewness and direction of skewness?

$\bar{x} >$ median right

$\frac{3(14.2 - 14)}{3.1}$

A. -0.193, indicating left skewness

B. 0.000, indicating perfect symmetry

C. 0.538, indicating moderate right skewness

D. 1.200, indicating strong right skewness

E. 0.193, indicating slight right skewness.

6. What is the IQR of these data?

$IQR = Q_3 - Q_1$

$= 15.5 - 12$

$Q_3 = 15.5$

$Q_1 = 12$

A. 3

B. 3.5

C. 4

D. 4.5

E. 5

6

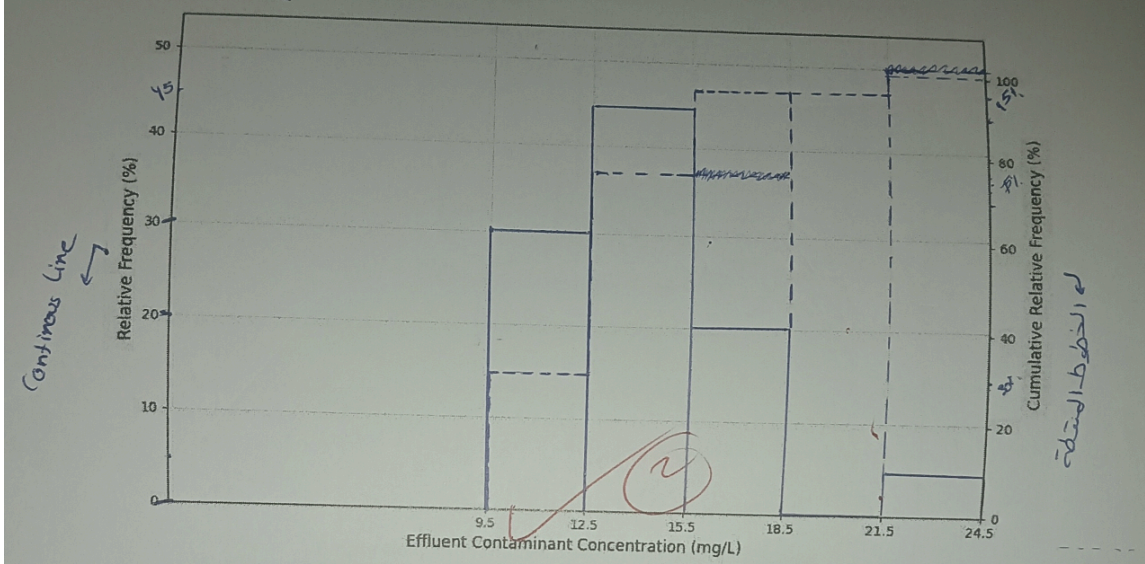
7. Fill in the following table according to the intervals shown

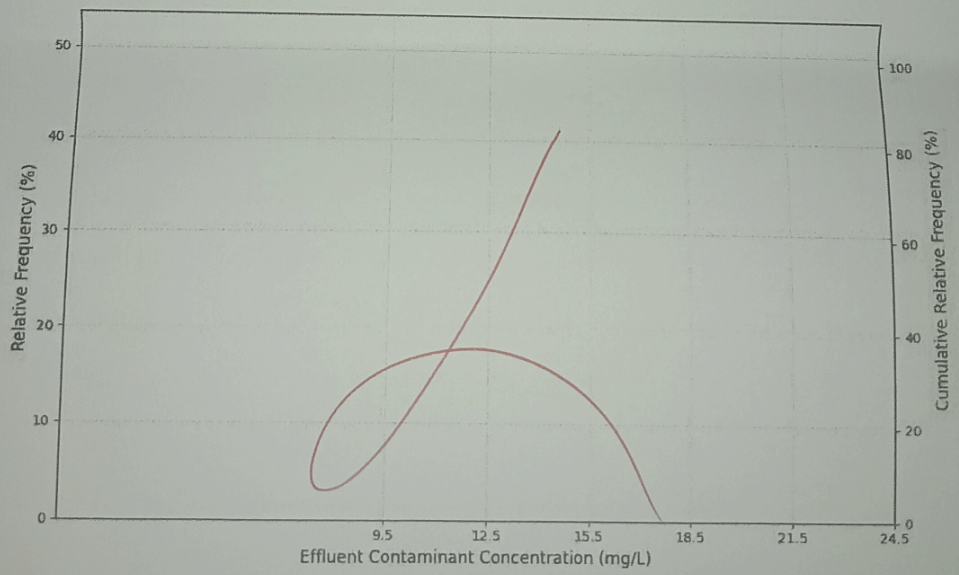
4

Interval	Frequency (mg/L)	Relative frequency (%)	Cumulative frequency (%)
[9.5, 12.5)	6	$\frac{6}{20} = 0.3 \times 100 = 30\%$	30%
[12.5, 15.5)	9	$\frac{9}{20} \times 100\% = 45\%$	$30\% + 45\% = 75\%$
[15.5, 18.5)	4	$\frac{4}{20} \times 100 = 20\%$	$75\% + 20\% = 95\%$
[18.5, 21.5)	0	$\frac{0}{20} \times 100 = 0\%$	$95\% + 0\% = 95\%$
[21.5, 24.5]	1	$\frac{1}{20} \times 100 = 5\%$	$95\% + 5\% = 100\%$
Sum	20	100%	

MADAR

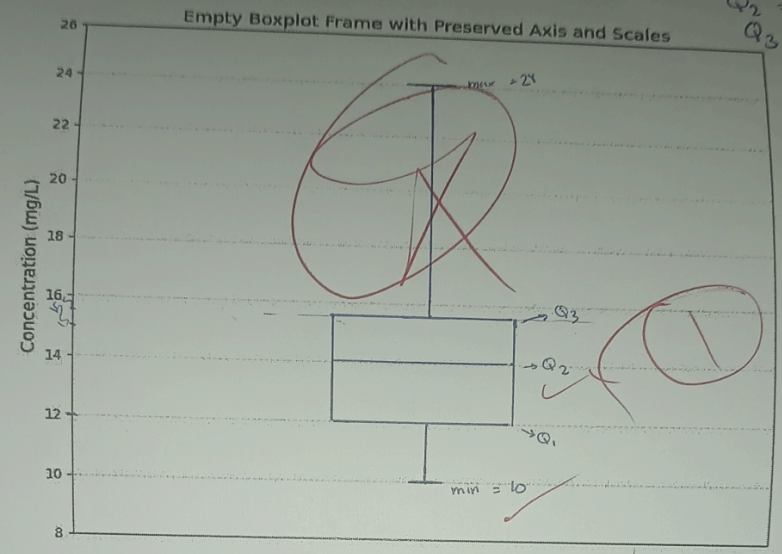
8. **Generate a histogram of relative frequency of these data with cumulative frequency overlaid on it.**

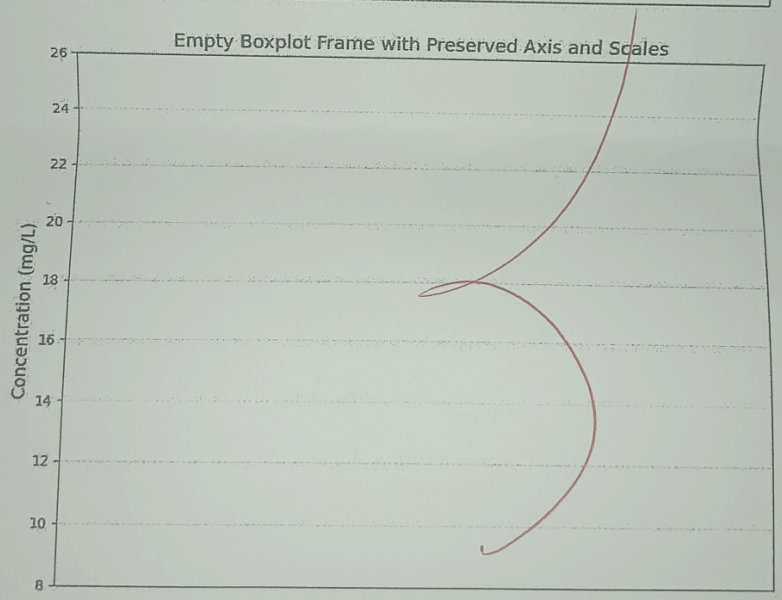




9. **Generate a boxplot of these data.**

min = 10
max = 24
Q_1 = 12
Q_2 = median = 14
Q_3 = 15.5





2. (5 Marks) A plant recorded the following 3-by-3 contingency table for 200 batches. The two variables are mixing intensity and product particle-size classification.

Mixing intensity	Fine	Standard	Coarse	Row total
Low	10	20	10	40
Medium	15	45	20	80
High	5	35	40	80
Column total	30	100	70	200

Let L denote low mixing, M denote medium mixing, and H denote high mixing.
Let F denote fine particles, S denote standard particles, and C denote coarse particles.

10. What is $P(L \cap F)$?
- A. 0.05
- B. 0.10
- C. 0.15
- D. 0.20
- E. 0.30

11. What is $P(M \cup C)$?
- A. 0.45
- B. 0.55
- C. 0.70
- D. 0.65
- E. 0.75

12. What is $P(H')$?
- A. 0.20
- B. 0.35
- C. 0.40
- D. 0.80
- E. 0.60

13. What is $P(F' \cap H)$?
- A. 0.025
- B. 0.175
- C. 0.375
- D. 0.300
- E. 0.425

14. What is $P(L' \cup F)$?
- A. 0.15
- B. 0.85
- C. 0.55
- D. 0.70
- E. 0.95

3. *(6 Marks)* A specialty chemical plant produces a high-purity solvent by continuously blending outputs from four parallel distillation columns, D_1, D_2, D_3, and D_4. Historical production records show that each column contributes a different fraction of the total daily output. Each column also has a different probability of producing off-specification material.

Column	Fraction of total production	Probability of off-spec output
D_1	0.35	0.01
D_2	0.25	0.04
D_3	0.20	0.07
D_4	0.20	0.12

During routine quality control, a randomly selected batch from the blended storage tank is found to be off-specification. PLEASE SHOW YOUR WORK ON THE OPPOSITE PAGE, NO MARKS WILL BE GIVEN WITHOUT DETAILS OF YOUR WORK.

15. What is the overall probability that a randomly selected batch is off-specification?

A. 0.0395
B. 0.0460
C. 0.0515
D. 0.0575
E. 0.0630

$P(O) = (0.01)(0.35) + (0.04)(0.25) + (0.07)(0.2) + (0.12)(0.2) = 0.0515$

16. What is $P(D_3 \mid \text{Off})$?

A. 0.0679
B. 0.1942
C. 0.3425
D. 0.2718
E. 0.4660

$P(D_3 \mid Off) = \frac{P(D_3 \cap Off)}{P(O)} = \frac{(0.2)(0.07)}{0.0515} = 0.2718$

17. Which column is the most likely source of the off-specification batch?

A. D_1
B. D_2
C. D_3
D. D_4
E. Cannot be determined

$P(D_3) \rightarrow 0.2718$
$P(D_1) \rightarrow 0.068$
$P(D_2) \rightarrow 0.194$
$P(D_4) \rightarrow 0.466$

MADAR

4. **(5 marks) In a semi-batch reactor with intermittent mixing, the effective residence time τ, in minutes, of fluid elements is observed to take four discrete values depending on the mixing regime. Historical monitoring gives the following probability distribution:**

Residence time τ in minutes	$X = 1 - e^{-0.2\tau}$	Probability	Mixing regime
1	$X = 0.18$	0.10	Poor mixing
3	$X = 0.45$	0.40	Moderate mixing
5	$X = 0.632$	0.30	Good mixing
7	$X = 0.753$	0.20	Excellent mixing

A first-order irreversible reaction $A \rightarrow B$ occurs in the reactor with rate constant

$$k = 0.20 \text{ min}^{-1}.$$

The conversion of reactant A for a fluid element with residence time τ is

$$X(\tau) = 1 - e^{-k\tau}.$$

Since τ is a discrete random variable, the conversion X is also a discrete random variable.

a) Determine the probability mass function of X?

$\tau = 1 \rightarrow X = 1 - e^{-0.2(1)}$
$= 0.18$

$\tau = 3 \rightarrow X = 1 - e^{-0.2(3)}$
$= 0.45$

$\tau = 5 \rightarrow X = 1 - e^{-0.2(5)}$
$= 0.63$

$\tau = 7 \rightarrow X = 1 - e^{-0.2(7)}$
$= 0.75$

$X(\tau)$	Probability	$P(X(\tau))$
0.18	0.1	0.018
0.45	0.4	0.18
0.632	0.3	0.1896
0.753	0.2	0.1506

b) Determine the expected conversion.

$E(X) = \sum X f(X)$

$= (0.18)(0.1) + (0.45)(0.4) + (0.632)(0.3) + (0.753)(0.2)$

$E(X) = 0.5382$

5

MADAR