Process Heat Transfer

Lec 6: Unsteady-State Conduction

Introduction, Lumped-Heat-Capacity System, Biot Number

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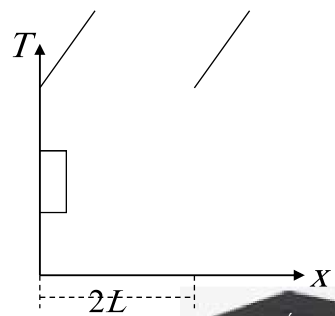
- Lumped-Heat-Capacity System
- Applicability of Lumped-Capacity Analysis

Introduction

- A heat transfer process for which the temperature varies with time, as well as location within a solid.
- It is initiated whenever a system experiences a change in operating conditions and proceeds until a new steady state (thermal equilibrium) is achieved.
- It can be induced by changes in:
  - surface convection conditions (h, T_∞),
  - surface radiation conditions (h_r, T_{surr}),
  - a surface temperature or heat flux, and/or
  - internal energy generation.
- As an example, If a solid immersed in an environment, time elapsed to attain equilibrium.

Introduction

- So far, we have only considered the temperature distribution and calculation of heat transfer at equilibrium or steady state.
- It is desired some times to determine the required time to attain equilibrium, or that needed to attain certain temperature at certain point of the body.
  - This is important for the design of large numbers of heating and cooling processes
- This can be achieved by solving the heat-conduction equation which considers the time variation of the temperature
- Consider the finite plate shown with thickness $2L$.
  - ✓ Initially the plate is at uniform temperature T_i
  - ✓ At t = 0, the surfaces are suddenly lowered to $T = T_1$



What is $T = T(x, t)$?

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Introduction

- Differential equation describing this non-heat source, one-dimensional problem is

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

- PDF in x and t
- Requires 2 boundary conditions and 1 initial condition

- The final series solution is

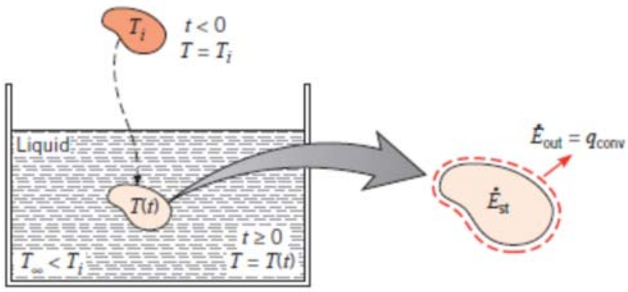
$$\frac{\theta}{\theta_i} = \frac{T - T_1}{T_i - T_1} = \frac{4}{\pi}\sum_{n=1}^{\infty} \exp\left\{-\left(\frac{n\pi}{2L}\right)^2 \alpha t\right\} \sin\left(\frac{n\pi}{2L}x\right)$$

Lumped-Heat-Capacity System

- One way of simplifying the approach to transient conduction problems would be to consider that the temperature in the solid body varies with time but at any instant T does not vary with position, i.e. temperature gradients within the solid are negligible.
- That is, T at all locations inside the solid varies uniformly with time.
- If we assume that the energy transferred from the solid is removed by convection to a fluid, the condition for a uniformly varying temperature within the solid would be satisfied when the resistance to conduction within the solid is much less than the resistance to convection between the solid and its surroundings
- This is referred to system with ***negligible internal resistance*** or ***Lumped Capacitance System***

Lumped-Heat-Capacity System

- The convection heat loss from the body is evidenced as a decrease in the internal energy of the body. Thus,



$$-\dot{E}_{out} = \dot{E}_{st}$$

$$\dot{Q} = hA(T - T_\infty) = -C\rho V \frac{dT}{dt}$$

where A is the surface area for convection and V is the volume

I.C. @ $t = 0$ $T = T_o$ ⇨

$$\frac{T - T_\infty}{T_o - T_\infty} = e^{-[hA/\rho CV]t}$$

where $\frac{1}{hA}\rho CV = \tau$ thermal time constant

Lumped-Heat-Capacity System

or $R_t C_t = \tau$

R_t is the resistance to convection heat transfer

C_t is the *lumped thermal capacitance* of the solid

$$R_t = \frac{1}{hA} \qquad C_t = \rho CV$$

Assuming that $\theta \equiv T - T_\infty$

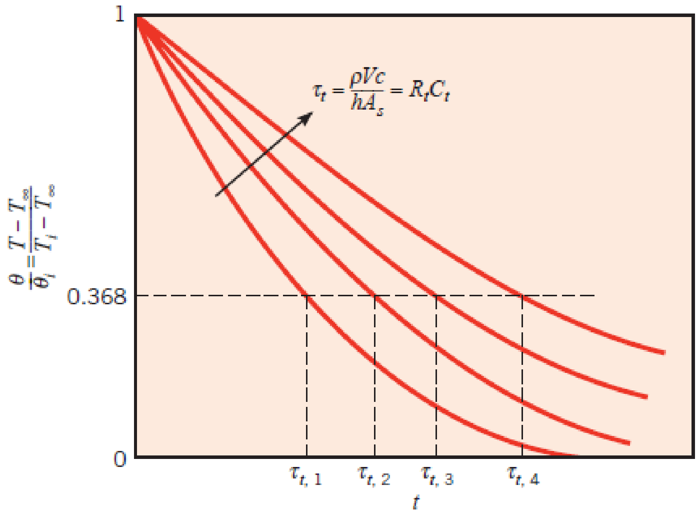
⇨

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{hA_s}{\rho Vc}\right)t\right]$$

and $\frac{\rho Vc}{hA_s} \ln \frac{\theta_i}{\theta} = t$

✓ When $t = 1\tau$ $\Rightarrow (T - T_\infty) = 0.368(T_o - T_\infty)$

Lumped-Heat-Capacity System



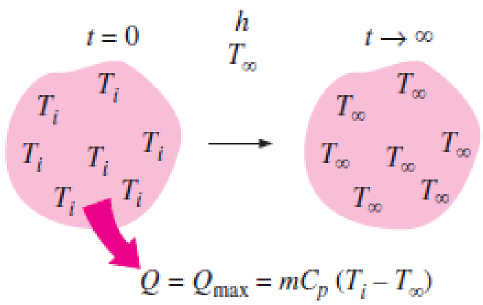
Lumped-Heat-Capacity System

Heat Transfer rate

- Once the temperature $T(t)$ at time t is available, the *rate* of convection heat transfer between the body and its environment at that time can be determined

$$\dot{Q}(t) = hA_s[T(t) - T_\infty] \qquad \text{(W)}$$

- The amount of heat transfer reaches its *upper limit* when the body reaches the surrounding temperature T_∞.
- Therefore, the *maximum* heat transfer between the body and its surroundings is



$$Q_{max} = mC_p(T_\infty - T_i) \qquad \text{(kJ)}$$

Lumped-Heat-Capacity System

➢ To determine the total energy transfer Q occurring up to some time t

$$Q = \int_0^t q\,dt = hA_s \int_0^t \theta\,dt$$

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{hA_s}{\rho V c}\right)t\right]$$

$$\Rightarrow \quad Q = (\rho V c)\theta_i \left[1 - \exp\left(-\frac{t}{\tau_t}\right)\right]$$

or $$Q = mC_p[T(t) - T_i] \qquad \text{(kJ)}$$

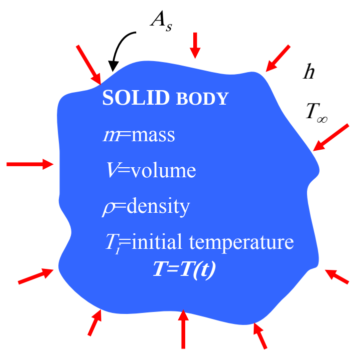
Applicability of Lumped-Capacity Analysis

➢ Lumped-capacity analysis, assumes that:

surface-convective resistance >>> internal-coduction resistance (valid assumption for low values of *h*)

➢ It is expected to yield reasonable estimates within about 5% when:

$$\frac{h(V/A)}{k} < 0.1$$

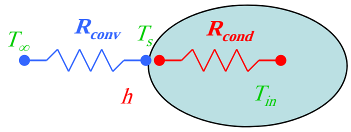


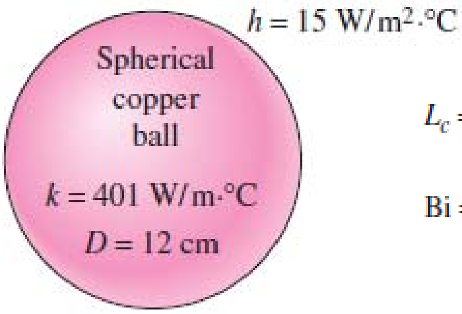
If $$l_c = \frac{V}{A}$$

Then $$\frac{hl_c}{k} = \text{Bi} < 0.1$$ ⇒ Necessary condition for the application of the lumped-capacity model

Applicability of Lumped-Capacity Analysis

$$Bi = \frac{L_c / k}{1 / h} = \frac{R_{cond}}{R_{conv}} = \frac{\text{Conduction resistance within the body}}{\text{Convection resistance at the surface of the body}}$$

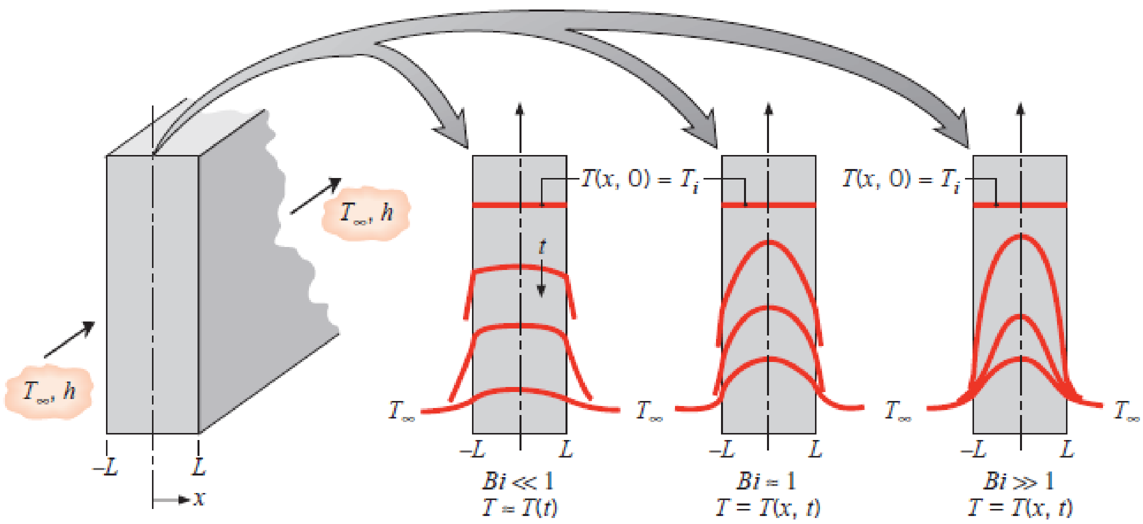




$$L_c = \frac{V}{A_s} = \frac{\frac{1}{6}\pi D^3}{\pi D^2} = \frac{1}{6} D = 0.02 \text{ m}$$

$$\text{Bi} = \frac{hL_c}{k} = \frac{15 \times 0.02}{401} = 0.00075 < 0.1$$

Applicability of Lumped-Capacity Analysis

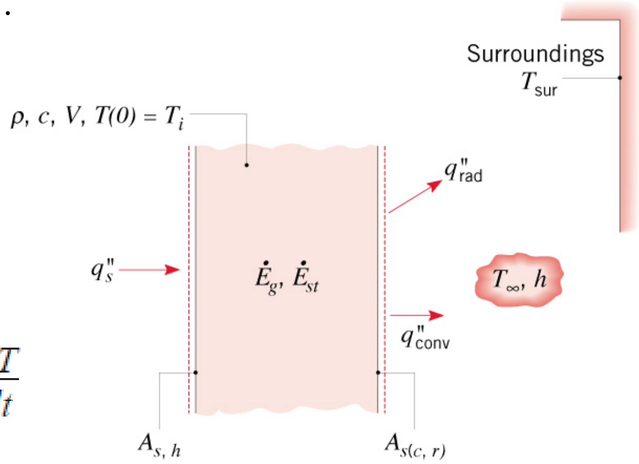


lumped-capacity system

General Analysis

- Based on the assumption of a spatially uniform temperature distribution throughout the transient process. Hence, $T(r,\vec{t}) \approx T(t)$.

➢ Consider a general case, which includes convection, radiation and/or an applied heat flux at specified surfaces as well as internal energy generation



$$q''_s A_{s,h} + \dot{E}_g - (q''_{conv} + q''_{rad}) A_{s(c,r)} = \rho V c \frac{dT}{dt}$$

$$\Rightarrow \quad q''_s A_{s,h} + \dot{E}_g - [h(T - T_\infty) + \varepsilon\sigma(T^4 - T^4_{sur})] A_{s(c,r)} = \rho V c \frac{dT}{dt}$$

General Analysis

If only radiation is included

$$\rho V c \frac{dT}{dt} = -\varepsilon A_{s,r} \sigma (T^4 - T^4_{sur})$$

$$\Rightarrow \quad t = \frac{\rho V c}{4 \varepsilon A_{s,r} \sigma T^3_{sur}} \left\{ \ln \left| \frac{T_{sur} + T}{T_{sur} - T} \right| - \ln \left| \frac{T_{sur} + T_i}{T_{sur} - T_i} \right| + 2 \left[\tan^{-1}\left(\frac{T}{T_{sur}}\right) - \tan^{-1}\left(\frac{T_i}{T_{sur}}\right) \right] \right\}$$

If radiation is neglected

If $\theta \equiv T - T_\infty$, $a \equiv (h A_{s,c} / \rho V c)$ and $b \equiv [(q''_s A_{s,h} + \dot{E}_g) / \rho V c]$.

General Analysis

$$\frac{d\theta}{dt} + a\theta - b = 0$$

Introduce $\theta' \equiv \theta - \frac{b}{a}$ $\Rightarrow$ $\frac{d\theta'}{dt} + a\theta' = 0$

And hence $\frac{\theta'}{\theta'_i} = \exp(-at)$

$$\Rightarrow \quad \frac{T - T_\infty}{T_i - T_\infty} = \exp(-at) + \frac{b/a}{T_i - T_\infty}[1 - \exp(-at)]$$

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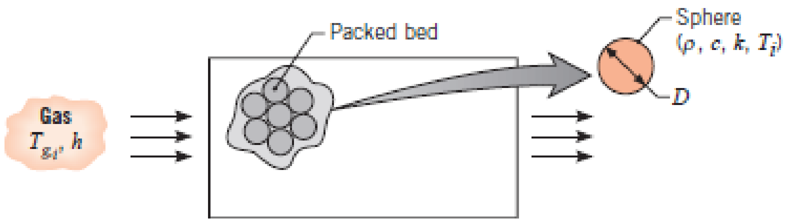
Example

Consider a packed bed of 75-mm-diameter aluminum spheres ($\rho = 2700\ \text{kg/m}^3$, $c = 950\ \text{J/kg}\cdot\text{K}$, $k = 240\ \text{W/m}\cdot\text{K}$) and a charging process for which gas enters the storage unit at a temperature of $T_{g,i} = 300°\text{C}$. If the initial temperature of the spheres is $T_i = 25°\text{C}$ and the convection coefficient is $h = 75\ \text{W/m}^2\cdot\text{K}$, how long does it take a sphere near the inlet of the system to accumulate 90% of the maximum possible thermal energy? What is the corresponding temperature at the center of the sphere? Is there any advantage to using copper instead of aluminum?



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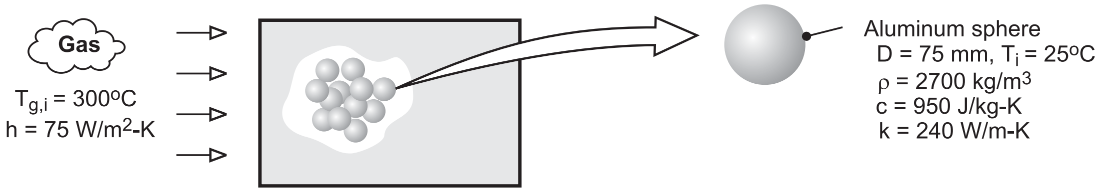
Example cont.

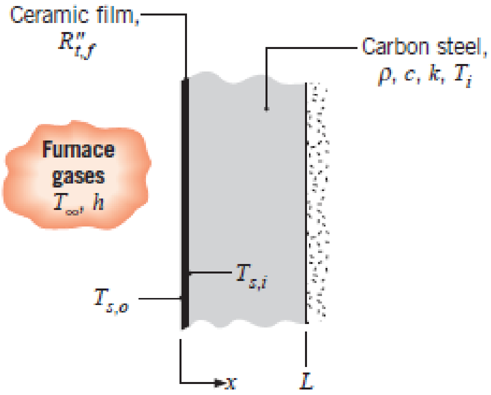
Schematic:



Example cont.

Example

5.22 A plane wall of a furnace is fabricated from plain carbon steel ($k = 60$ W/m·K, $\rho = 7850$ kg/m^3, $c = 430$ J/kg·K) and is of thickness $L = 10$ mm. To protect it from the corrosive effects of the furnace combustion gases, one surface of the wall is coated with a thin ceramic film that, for a unit surface area, has a thermal resistance of $R''_{t,f} = 0.01$ m^2·K/W. The opposite surface is well insulated from the surroundings.



At furnace start-up the wall is at an initial temperature of $T_i = 300$ K, and combustion gases at $T_\infty = 1300$ K enter the furnace, providing a convection coefficient of $h = 25$ W/m^2·K at the ceramic film. Assuming the film to have negligible thermal capacitance, how long will it take for the inner surface of the steel to achieve a temperature of $T_{s,i} = 1200$ K? What is the temperature $T_{s,o}$ of the exposed surface of the ceramic film at this time?

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Example cont.

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Example cont.

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