# The University of Jordan

School of Engineering

Department of Chemical Engineering

Chemical Engineering Laboratory (4) (0905562)

Section no. (1)

Experiment no. (5)

Development of Kinetic Rate Equation from Hydraulic Analog Methods

Short Report

Date of Submission: 3rd December 2023

Student Name	Registration #
Sedra Nibal Al-Burghili	0191293
Nour Ahmad Kharisat	0194083
Donia Majdi Khalil	0197045
Sura Mohammad Froukh	0197130
Lana Mamoon Abdelnabi	0198593

Instructor: Eng. Nariman Al-Dahleh
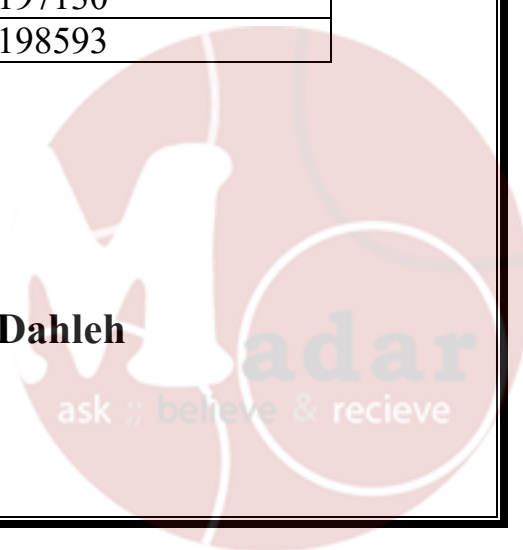
ABSTRACT

The experiment focused on the development of a hydraulic analog method to derive kinetic rate equations. The modular design of the hydraulic analog allowed for simulating linear and nonlinear reactions using a burette and a capillary tube. For a first-order irreversible reaction, the rate equation was determined as $-r_A = 0.1469C_A$. In the case of a first-order series reaction, the rate equations were $-r\_A = 0.217C_A$ and $-r_S = 0.1055C_R$ for a first-order reversible reaction, the rate equation was $-r_A = 0.023C_A - 0.009C_R$. Nonlinear reactions were also studied using a funnel, and the rate equations were $-r_A = 0.604C_A^{0.65}$ and $-r_A = 0.217C_A^{0.6615}$ for n > 1. It should be noted that obtaining an order below first order for a reaction may be uncommon and could be attributed to experimental limitations or errors in the linearization method.
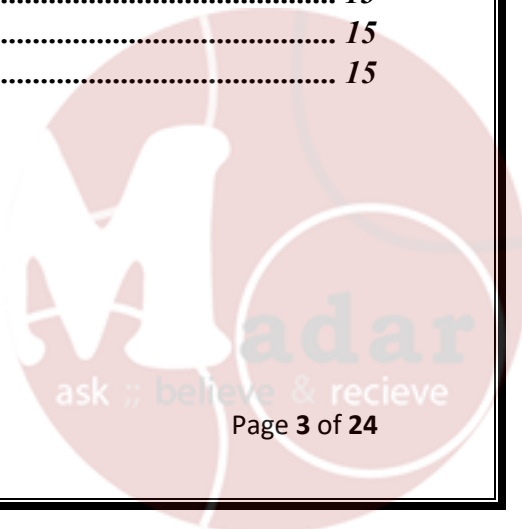
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1. First Order Irreversible Reaction

Table 1: Experimental and calculated data for first order irreversible reaction

Time (s)	Cumulative Time (s)	Cumulative Time (min)	Volume (ml)	$\ln(V/V_0)$
0	0	0.00	50	0.00
11	11	0.18	49	0.02
10	21	0.35	48	0.04
13	34	0.57	47	0.06
10	44	0.73	46	0.08
11	55	0.92	45	0.11
11	66	1.10	44	0.13
13	79	1.32	43	0.15
14	93	1.55	42	0.17
11	104	1.73	41	0.20
13	117	1.95	40	0.22
14	131	2.18	39	0.25
11	142	2.37	38	0.27
12	154	2.57	37	0.30
11	165	2.75	36	0.33
14	179	2.98	35	0.36
14	193	3.22	34	0.39
13	206	3.43	33	0.42
14	220	3.67	32	0.45
15	235	3.92	31	0.48
15	250	4.17	30	0.51
16	266	4.43	29	0.54
16	282	4.70	28	0.58
17	299	4.98	27	0.62
19	318	5.30	26	0.65
15	333	5.55	25	0.69
18	351	5.85	24	0.73
20	371	6.18	23	0.78
21	392	6.53	22	0.82
20	412	6.87	21	0.87
20	432	7.20	20	0.92
23	455	7.58	19	0.97
23	478	7.97	18	1.02
24	502	8.37	17	1.08
25	527	8.78	16	1.14
28	555	9.25	15	1.20
29	584	9.73	14	1.27
31	615	10.25	13	1.35
30	645	10.75	12	1.43
35	680	11.33	11	1.51
35	715	11.92	10	1.61

38	753	12.55	9	1.71
41	794	13.23	8	1.83
44	838	13.97	7	1.97
47	885	14.75	6	2.12
47	932	15.53	5	2.30
64	996	16.60	4	2.53
75	1071	17.85	3	2.81
90	1161	19.35	2	3.22
96	1257	20.95	1	3.91
151	1408	23.47	0	-
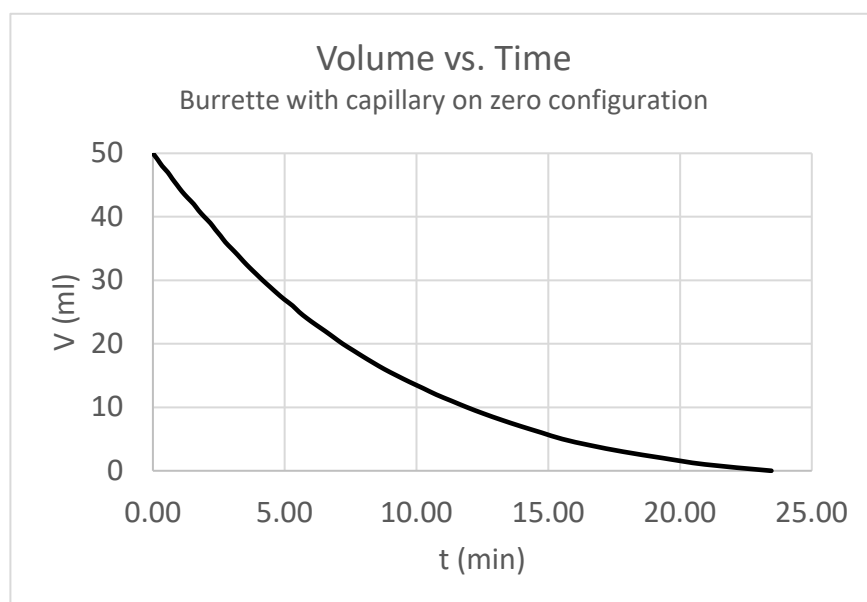
Figure 1: Volume versus Time for 1st order irreversible reaction

Slope (k)	0.1469
$C_A = 50e^{-0.1469t}$	
$r_A = 0.1469 C_A$	
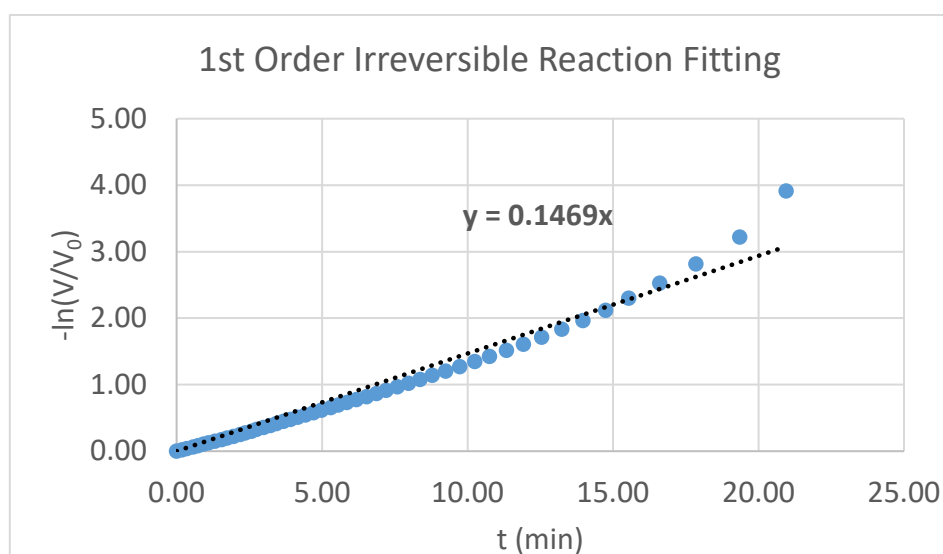
Figure 2: Data fitting on 1st order irreversible reaction equation

2. First Order Series Reaction

Table 2: Experimental and calculated data for series reactions

Time (min)	Volume A (ml)	Volume R (ml)	Volume S (ml)	$\ln(V_A/V_{A0})$	$\ln(V_R/V_{R4})$
0	47.7	0	0	0.000	
1	41.6	4.5	1.6	0.137	
2	35.9	7.2	4.6	0.284	
3	31.1	8.5	8.1	0.428	
4	26.7	8.8	12.2	0.580	0.000
5	22.7	8.5	16.5	0.743	0.035
6	19.5	8	20.2	0.895	0.095
7	16.6	7.5	23.6	1.056	0.160
8	14	6.6	27.1	1.226	0.288
9	11.7	5.5	30.5	1.405	0.470
10	9.7	5	33	1.593	0.565
11	8.1	4.5	35.1	1.773	0.671
12	6.5	3.9	37.3	1.993	0.814
13	5.2	3.4	39.1	2.216	0.951
14	4.1	2.8	40.8	2.454	1.145
15	3.2	2.5	42	2.702	1.258
16	2.4	2	43.3	2.989	1.482
17	1.7	1.75	44.25	3.334	1.615
18	1.1	1.5	45.1	3.770	1.769
19	0.4	1.1	46.2	4.781	2.079
20	0.2	1	46.5	5.474	2.175
21	0	0.5	47.2		2.868
22	0	0.5	47.2		2.868
23	0	0.2	47.5		3.784
24	0	0	47.7		
25	0	0	47.7		
26	0	0	47.7		
27	0	0	47.7		
28	0	0	47.7		
29	0	0	47.7		
30	0	0	47.7		
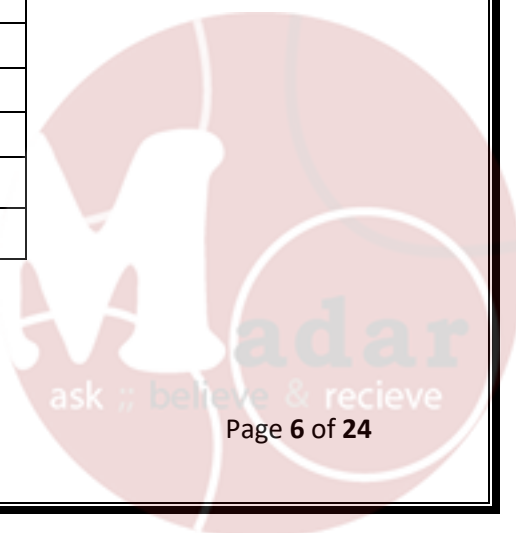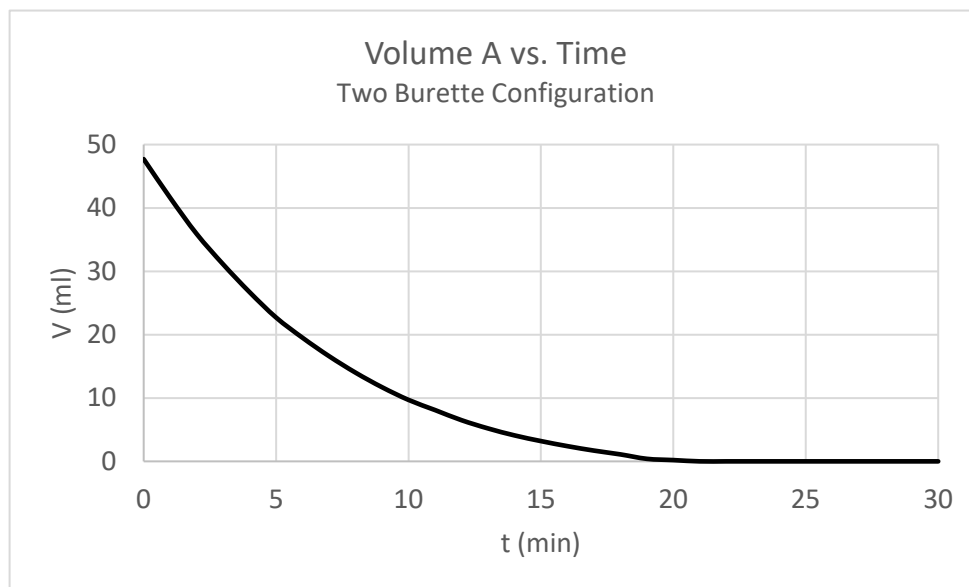
Figure 1: Volume of A vs. time for series reactions

Slope (k_1)	0.217

Figure 2: k1 determination for series reactions fitting.
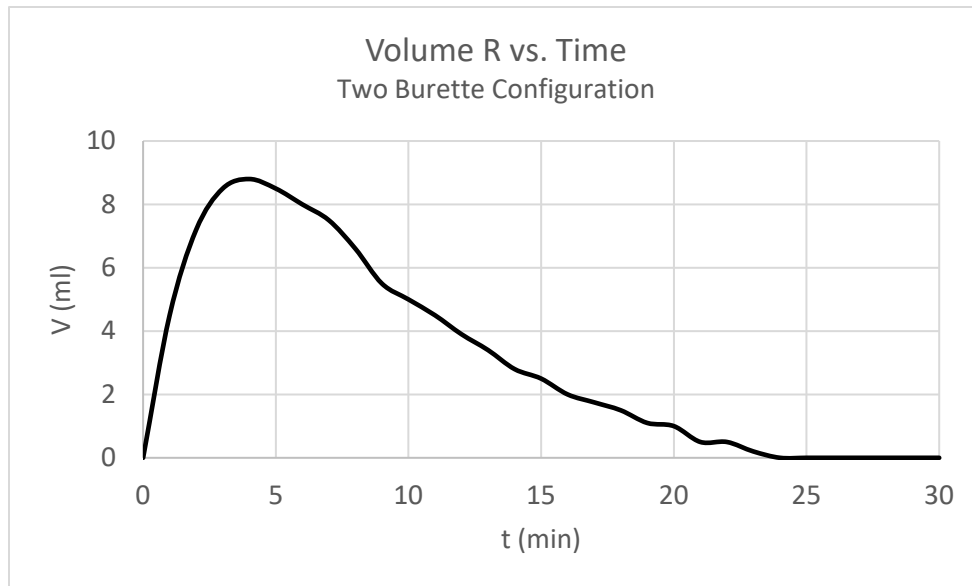
Figure 3: Volume of B vs. time for series reactions

→To obtain k2, the above figure from the peek to the tail is fitted in a first order irreversible reaction equation.

Slope (k$_2$)	0.1055
r$_A$=0.2017C$_A$	
r$_S$=0.106C$_R$	

Figure 4: k2 determination for series reactions fitting

Figure 5: Concentration of A,R & S vs. time for series reactions

3. First Order Reversible Reaction

Table 3: Experimental and calculated data for reversible reaction

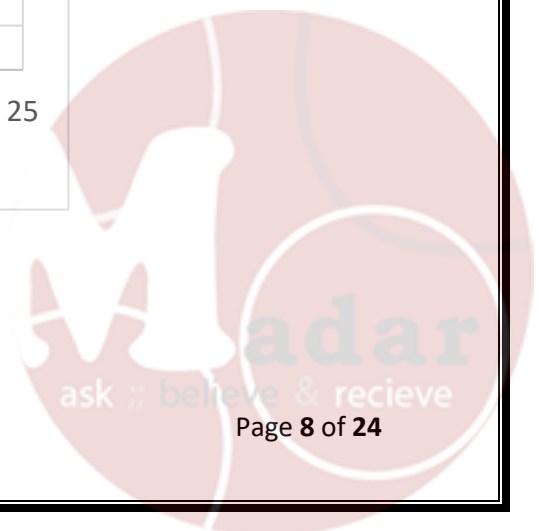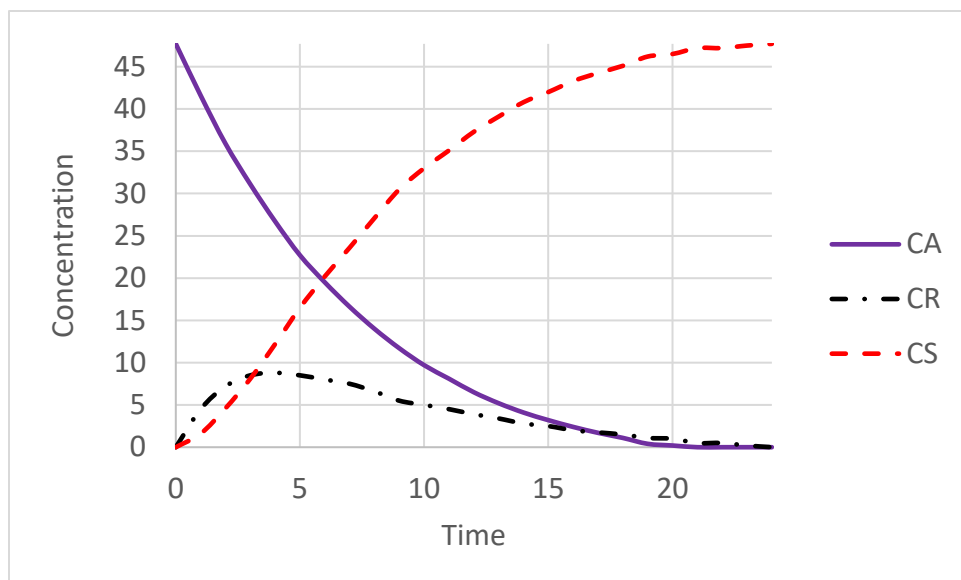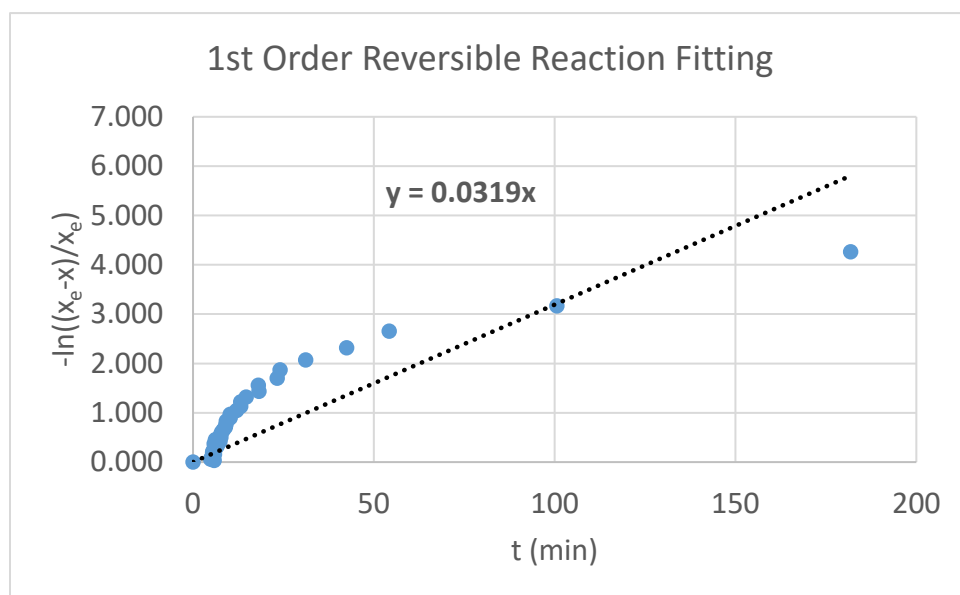| Time (s)	Cumulative Time (s)	Cumulative Time (min)	Volume (ml)	x	$\ln((x_e-x)/x_e)$
0	0	0.00	50	0	0.000
5.84	5.84	0.10	49	0.02	0.029
4.84	10.68	0.18	48	0.04	0.058
5.17	15.85	0.26	47	0.06	0.088
5.5	21.35	0.36	46	0.08	0.120
5.9	27.25	0.45	45	0.1	0.152
5.42	32.67	0.54	44	0.12	0.185
5.47	38.14	0.64	43	0.14	0.220
5.9	44.04	0.73	42	0.16	0.255
6.35	50.39	0.84	41	0.18	0.292
6.72	57.11	0.95	40	0.2	0.331
5.85	62.96	1.05	39	0.22	0.371
7.46	70.42	1.17	38	0.24	0.413
6.21	76.63	1.28	37	0.26	0.456
7.69	84.32	1.41	36	0.28	0.501
7.61	91.93	1.53	35	0.3	0.549
7.89	99.82	1.66	34	0.32	0.599
8.39	108.21	1.80	33	0.34	0.652
8.94	117.15	1.95	32	0.36	0.707
9.16	126.31	2.11	31	0.38	0.766
9.23	135.54	2.26	30	0.4	0.829
10.31	145.85	2.43	29	0.42	0.895
10.31	156.16	2.60	28	0.44	0.967
12.01	168.17	2.80	27	0.46	1.044
13.17	181.34	3.02	26	0.48	1.127
13.17	194.51	3.24	25	0.5	1.218
14.68	209.19	3.49	24	0.52	1.318
18.27	227.46	3.79	23	0.54	1.429
18.09	245.55	4.09	22	0.56	1.555
23.28	268.83	4.48	21	0.58	1.698
24.08	292.91	4.88	20	0.6	1.865
31.19	324.1	5.40	19	0.62	2.065
42.5	366.6	6.11	18	0.64	2.317
54.21	420.81	7.01	17	0.66	2.653
100.62	521.43	8.69	16	0.68	3.164
181.87	703.3	11.72	15	0.7	4.263
135.54	838.84	13.98	14.5	0.71	-

Figure 6: Volume vs. time for reversible reaction
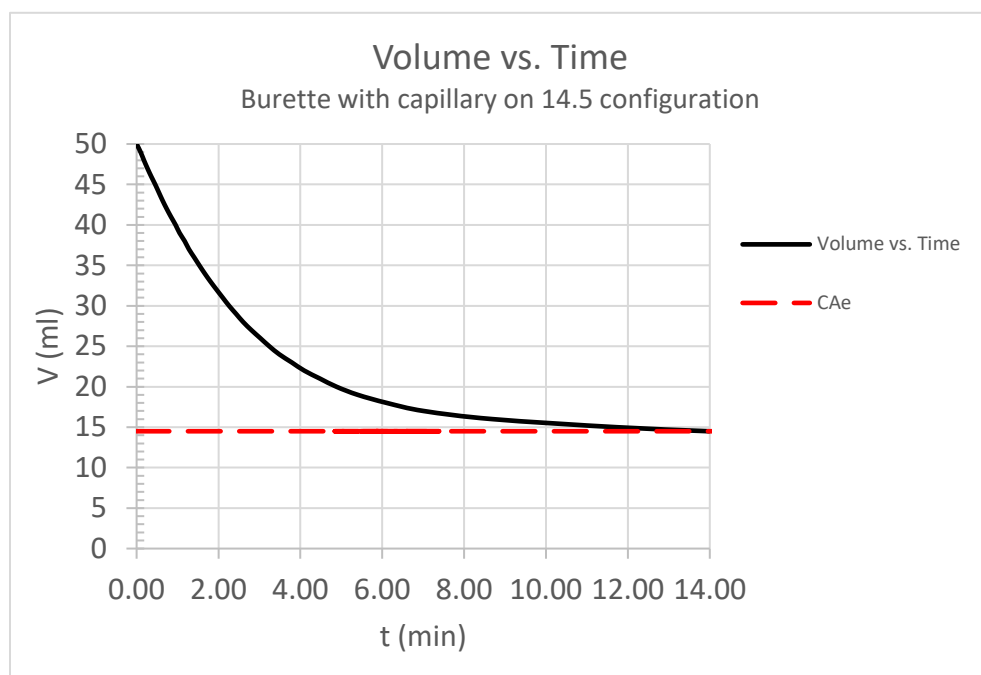
C_{A0}	50
C_{Ae}	14.5
x_e	0.71

Figure 7: Data fitting on reversible reaction equation

Slope	0.0319
k_1	0.022649
k_2	0.009251
$r_A=0.023C_A-0.009C_R$	
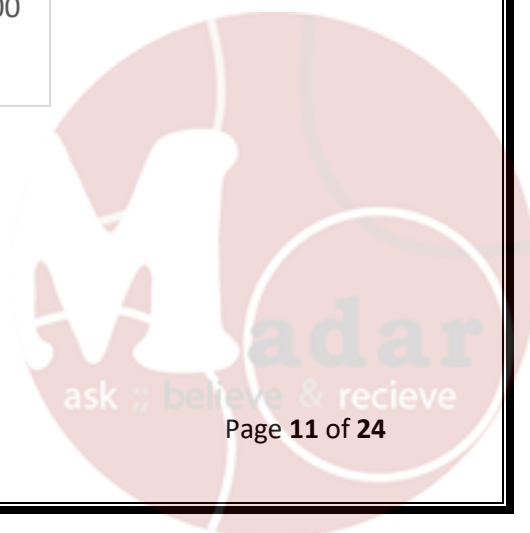
4. **Non-linear Reaction with Order Less Than One**

Table 4: Experimental and calculated data for n<1 reaction

Cumulative Time (min)	Volume (ml)	V_2-V_1	t_2-t_1	$\ln(-\Delta V/\Delta t)$	$\ln(V)$
0.00	50	0	0.00	#DIV/0!	3.912
0.31	49	-1	0.31	1.184	3.892
0.45	48	-1	0.14	1.932	3.871
0.60	47	-1	0.14	1.936	3.850
0.75	46	-1	0.15	1.894	3.829
0.90	45	-1	0.15	1.885	3.807
1.05	44	-1	0.15	1.887	3.784
1.20	43	-1	0.15	1.882	3.761
1.36	42	-1	0.16	1.844	3.738
1.50	41	-1	0.14	1.964	3.714
1.66	40	-1	0.16	1.849	3.689
1.82	39	-1	0.16	1.828	3.664
1.98	38	-1	0.16	1.823	3.638
2.15	37	-1	0.17	1.791	3.611
2.31	36	-1	0.17	1.783	3.584
2.47	35	-1	0.16	1.840	3.555
2.64	34	-1	0.17	1.786	3.526
2.80	33	-1	0.16	1.824	3.497
2.97	32	-1	0.17	1.784	3.466
3.16	31	-1	0.19	1.678	3.434
3.34	30	-1	0.18	1.719	3.401
3.51	29	-1	0.17	1.750	3.367
3.69	28	-1	0.18	1.707	3.332
3.87	27	-1	0.18	1.731	3.296
4.04	26	-1	0.18	1.735	3.258
4.23	25	-1	0.18	1.700	3.219
4.43	24	-1	0.20	1.609	3.178
4.63	23	-1	0.20	1.613	3.135
4.83	22	-1	0.20	1.615	3.091
5.04	21	-1	0.21	1.545	3.045
5.25	20	-1	0.21	1.561	2.996
5.46	19	-1	0.21	1.554	2.944
5.68	18	-1	0.22	1.531	2.890
5.90	17	-1	0.23	1.490	2.833
6.15	16	-1	0.24	1.415	2.773
6.38	15	-1	0.23	1.468	2.708
6.61	14	-1	0.24	1.436	2.639
7.15	13	-1	0.53	0.629	2.565
7.42	12	-1	0.28	1.286	2.485
7.70	11	-1	0.28	1.289	2.398
8.01	10	-1	0.31	1.174	2.303
8.33	9	-1	0.32	1.130	2.197
8.69	8	-1	0.36	1.011	2.079

9.05	7	-1	0.35	1.041	1.946
9.45	6	-1	0.41	0.899	1.792
9.94	5	-1	0.48	0.730	1.609
10.48	4	-1	0.54	0.613	1.386
11.24	3	-1	0.76	0.274	1.099
13.45	2	-1	2.22	-0.796	0.693
16.16	1	-1	2.71	-0.996	0.000
18.99	0	-1	2.83	-1.041	-
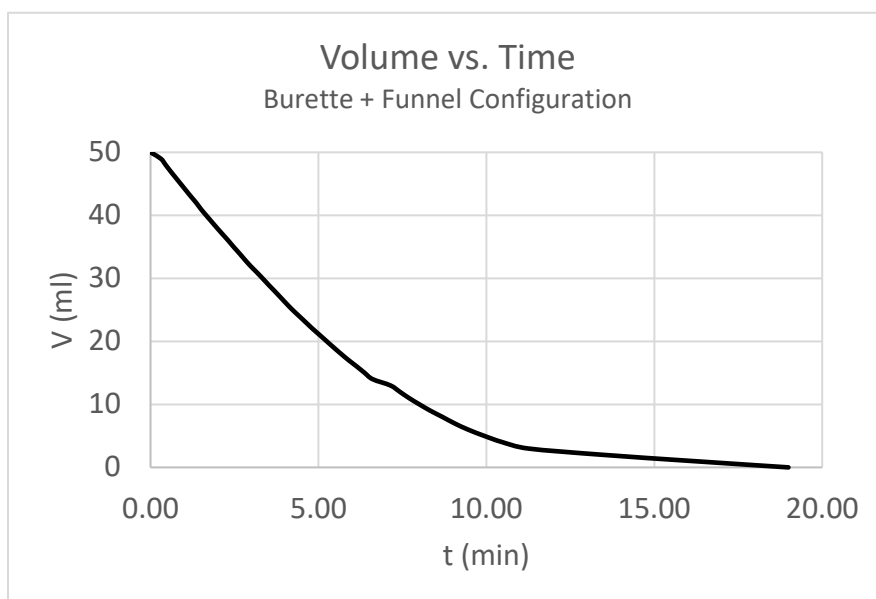
Figure 10: Volume vs. time for n<1 reaction
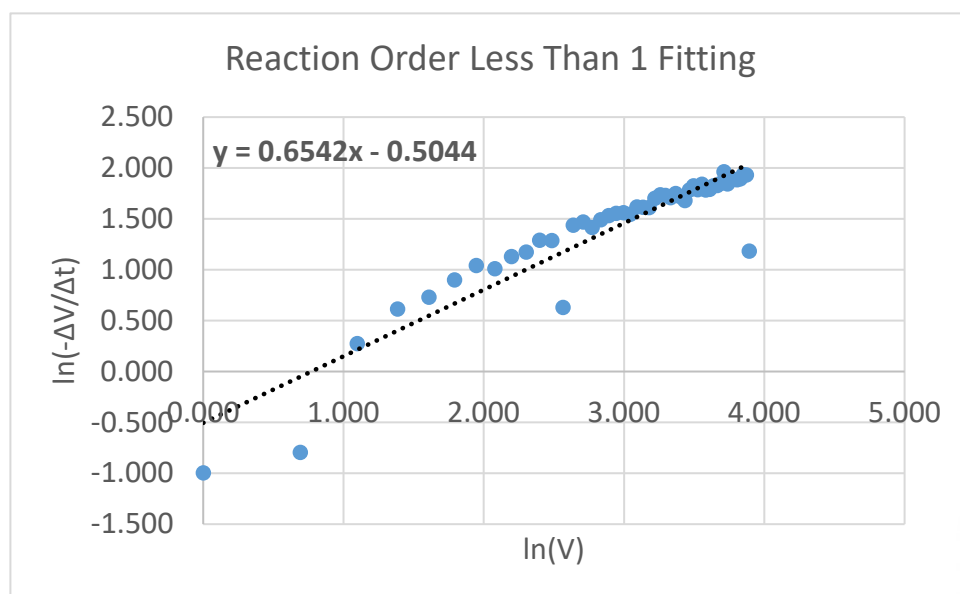
slope	0.6542
n	0.6542
intercept	-0.5044
k	0.60387
$r_A = 0.604 C_A^{0.65}$	

Figure 11: Data fitting on n<1 reaction equation
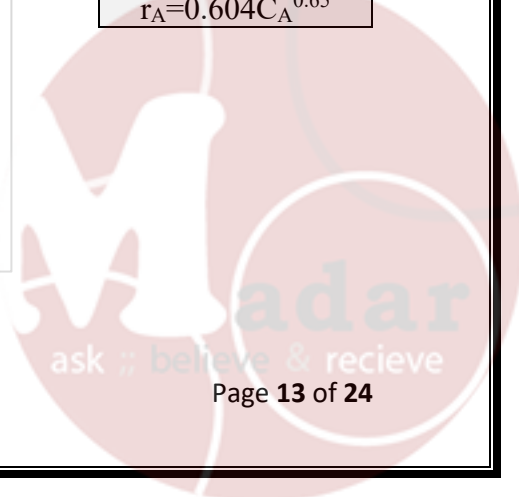
5. Non-linear Reaction with Order Higher Than One

Table 5: Experimental and calculated data for n>1 reaction

Cumulative Time (min)	Volume (ml)	V_2-V_1	t_2-t_1	$\ln(-\Delta V/\Delta t)$	$\ln(V)$
0.00	50	0	0.00	#DIV/0!	3.912
0.58	49	-1	0.58	0.553	3.892
0.91	48	-1	0.34	1.082	3.871
1.30	47	-1	0.39	0.942	3.850
1.65	46	-1	0.35	1.062	3.829
2.03	45	-1	0.38	0.976	3.807
2.40	44	-1	0.37	0.989	3.784
2.75	43	-1	0.35	1.059	3.761
3.13	42	-1	0.38	0.965	3.738
3.54	41	-1	0.42	0.871	3.714
3.93	40	-1	0.39	0.955	3.689
4.29	39	-1	0.36	1.019	3.664
4.66	38	-1	0.37	1.003	3.638
5.06	37	-1	0.40	0.912	3.611
5.47	36	-1	0.41	0.892	3.584
5.89	35	-1	0.42	0.873	3.555
6.32	34	-1	0.44	0.831	3.526
6.74	33	-1	0.42	0.879	3.497
7.18	32	-1	0.44	0.811	3.466
7.62	31	-1	0.44	0.818	3.434
8.08	30	-1	0.46	0.782	3.401
8.52	29	-1	0.44	0.826	3.367
9.04	28	-1	0.52	0.648	3.332
9.53	27	-1	0.49	0.714	3.296
10.04	26	-1	0.50	0.687	3.258
10.55	25	-1	0.51	0.667	3.219
11.08	24	-1	0.53	0.631	3.178
11.65	23	-1	0.57	0.569	3.135
12.97	22	-1	1.33	-0.284	3.091
13.38	21	-1	0.40	0.911	3.045
14.10	20	-1	0.72	0.324	2.996
14.66	19	-1	0.56	0.574	2.944
15.40	18	-1	0.74	0.302	2.890
16.11	17	-1	0.71	0.348	2.833
16.90	16	-1	0.79	0.230	2.773
17.72	15	-1	0.82	0.197	2.708
18.48	14	-1	0.76	0.280	2.639
19.28	13	-1	0.80	0.224	2.565
20.27	12	-1	0.99	0.006	2.485

21.24	11	-1	0.97	0.031	2.398
22.27	10	-1	1.03	-0.032	2.303
23.41	9	-1	1.13	-0.126	2.197
24.65	8	-1	1.24	-0.217	2.079
25.95	7	-1	1.30	-0.263	1.946
27.39	6	-1	1.44	-0.361	1.792
29.06	5	-1	1.68	-0.517	1.609
30.96	4	-1	1.90	-0.641	1.386
33.25	3	-1	2.28	-0.825	1.099
36.18	2	-1	2.94	-1.077	0.693
40.07	1	-1	3.89	-1.359	0.000
40.62	0	-1	0.55	0.603	-
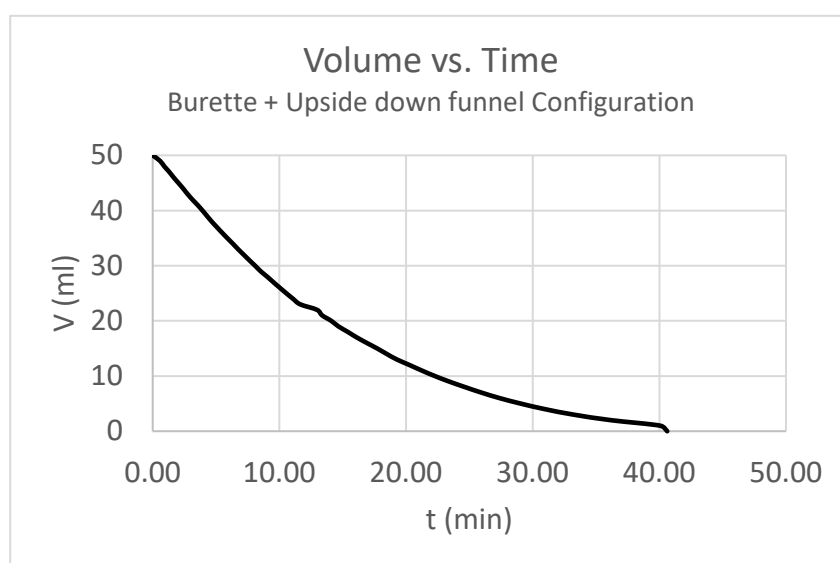
Figure 8: Volume vs. time for n>1 reaction

slope	0.6615
n	0.6615
intercept	-1.5287
k	0.21682
$rA=0.217CA^{0.6615}$	
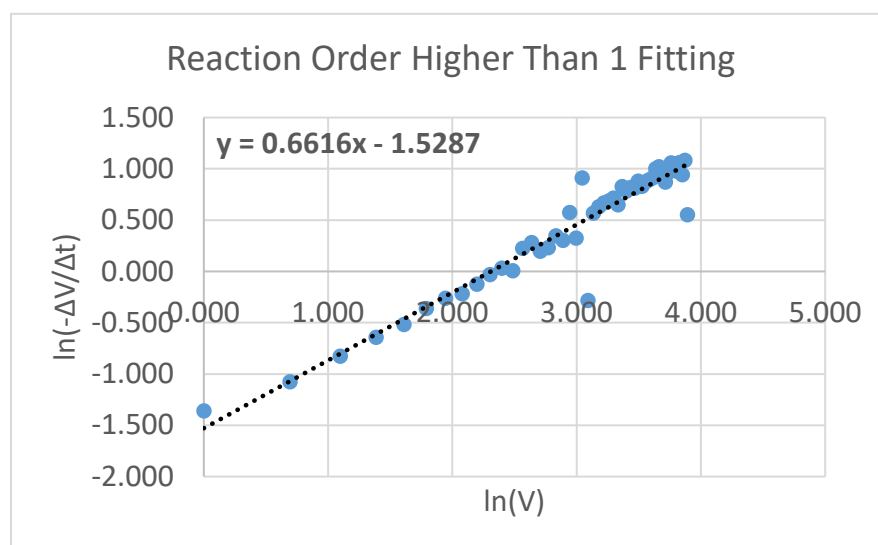
Figure 9: Data fitting on n>1 reaction equation
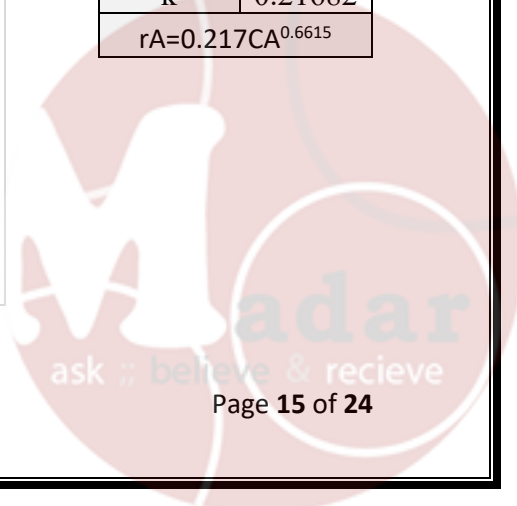
DISCUSSION

The hydraulic analog introduced has a modular design and can be composed, in its simplest form, of a burette to simulate linear reactions, or a funnel to simulate nonlinear reactions (first and second reactions) connected to a horizontally positioned capillary tube. Hydraulic experiment and its kinetic analog are reasonably represented by n^{th} order kinetics.

In this experiment, burette is used to present a batch reactor where liquids react **(water in this experiment simulates the reactions)**, change in volume (cm^3) inside the burette represents the change in concentration (mol/m^3) of component with time. Capillary tube is used to represent the reaction rate. Also, the position of capillary tube with respect to the burette volume level represents type of reaction; if it is at zero level, the reaction is reversible and if it is at higher level, the reaction is irreversible.

For first order irreversible reaction:

Burette is filled with a **Newtonian fluid (water)** and the liquid discharge-rate from the capillary is maintained low (in the form of drops which can be controlled by the diameter and length of the capillary) representing the decay of reactant concentration A to produce R. Plotting of **-ln (V/V$_0$)** versus **cumulative time should** give a straight line shown in figure (2). From this the reaction order and rate constant can be eliminated from the slope, which is equals to 0.1469, so the kinetic rate equation: $-r_A = 0.1469\ C_A$ as shown in table (1).

For first order series reaction:

The module can be extended into a series arrangement, to simulate a second order series reaction of the form: **(A→R→S)**. where the reaction takes a 2nd order behavior. Because of two step reaction: $-r_A = k_1\ C_A$ represents the rate of the first step, and $-r_S = k_2\ C_R$ is the rate of the second step, m≈n≈1.

The system is modeled using two burettes at different levels, liquid leaving the first one (decay in C_A) is filled into second one (Production of R), then drained from its bottom as decay of C_R to produce component S).

Plot **-ln (V$_A$/V$_{A0}$) vs. time** for first reaction, and **-ln (V$_R$/V$_{R4}$) vs. time** for the other one as shown in figure (4) & (6) respectively. The rate constants equal 0.217, 0.1055 respectively. Kinetic rate equations: $-r_A = 0.217 C_A$, $r_S = 0.1055 C_R$ as shown in table (2).

For first order reversible reaction:

The capillary tube is located at a level higher than zero level to ensure that there is no 100 % converging of A into R, some of R will go back to form A again, at the end of reaction, C_A is still more than zero (level of liquid does not equal to zero). **(A↔R).** A plot of **-ln ((x$_e$-x)/x$_e$)** versus **cumulative time should** give a straight line shown in figure (9). From this the reaction order and rate constants can be eliminated from the slope, which is equals to 0.0319, so the kinetic rate equation: $-r_A = 0.023 C_A - 0.009 C_R$ as shown in table (3).
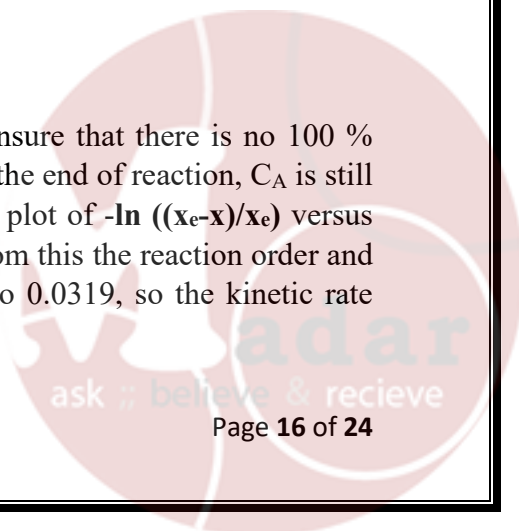
<u>For reactions with order not equal to one (Non-linear reactions):</u>

Non-linear reaction order can be obtained using a funnel. The reaction rate follows the form of equation, so we can linearize the equation.

For the case where $n < 1$, plotting **ln(-ΔV/Δt) vs. ln(V)** shown in figure (11). Getting the slope = 0.6542, so n = 0.6542 and intercept = -0.5044, then calculate k which is equal 8.005, so the kinetic rate equation: $-r_A = 0.604C_A^{0.65}$ as shown in table (4).

For the case where $n > 1$, plotting **ln(-ΔV/Δt) vs. ln(V)** shown in figure (13). Getting the slope = 0.6615, so n = 0.6615, Intercept = -1.5287, then calculate k which is equal 0.21681, so the kinetic rate equation: $-r_A = 0.217C_A^{0.6615}$ as shown in table (5). It's important to note that obtaining an order below 1st order for a reaction might be unusual and may indeed be due to experimental errors or limitations in the linearization method.

CONCLUSION AND RECOMMENDATIONS

- An analogy of a hydraulic system can be used to explain batch reactors that have reversible, irreversible, and series reactions, among other reaction types and orders.
- Reaction rate parameters can be obtained by integrated and differential approaches; the rate of reaction is dependent on reactant concentration.
- Examining the kinetic rate equation considering the observed variations in the reaction's order and researching the shift in the reaction's order at the rate constant are the primary objectives here.
- A burette is used to model linear reactions, whereas a funnel is used to simulate nonlinear reactions.
- In interactions that are irreversible, it ought to be zero. On the other hand, in interactions that are reversible, it must be greater than zero to prevent a 100% convergence of A into R.
- In this experiment, the reactant was water, and the volume readings in burette, expressed in cm^3, should be interpreted as the reactant's concentration, expressed in mol/m^3.
- Accuracy and taking the readings correctly are crucial to the successful completion of this experiment.
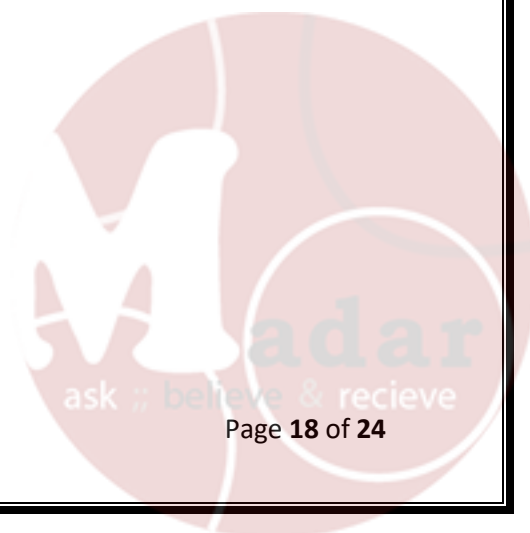
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2. H Scott Fogler (1999). Elements of chemical reaction engineering. Upper Saddle River, Nj: Prentice-Hall.
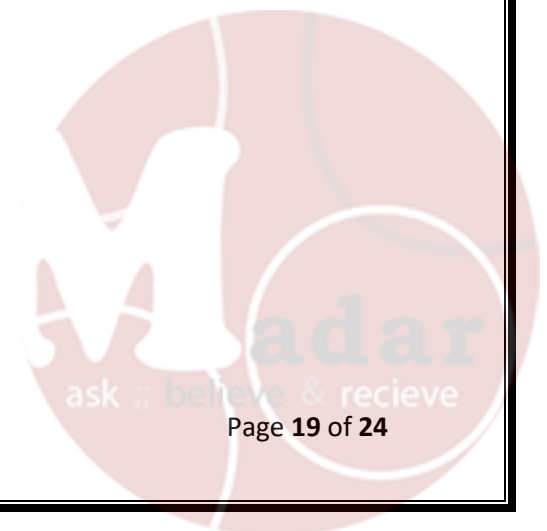
APPENDICES

I. Sample of calculations:

+ First order irreversible reaction $A \rightarrow R$

$$r_A = -kC_A{}^n$$

$$\frac{dC_A}{dt} = r_A = -kC_A{}^n \text{ and } n = 1$$

$$\int_{C_{Ao}}^{C_A} \frac{-dC_A}{C_A} = k \int_0^t dt$$

$$ln \frac{C_{Ao}}{C_A} = kt$$

$$C_A = 50e^{-0.1469t}$$

$$C_{Ao} = 50e^{-0.1469*0} = 50 \frac{mol}{m^3}$$

$$C_A = 49 \frac{mol}{m^3}$$

$$ln \frac{50}{49} = 0.02$$

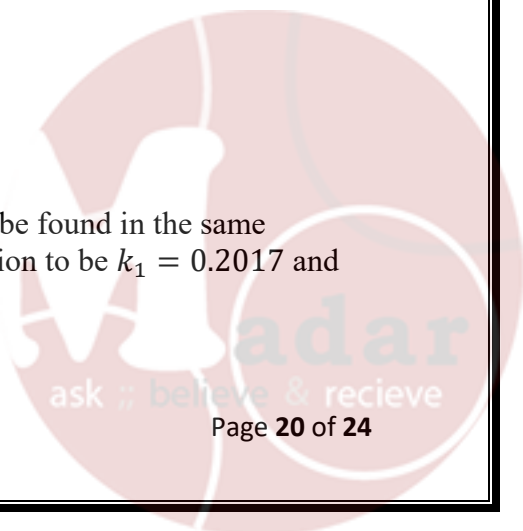Plot $ln \frac{C_{Ao}}{C_A}$ vs time for all concentration to get
Slope = 0.1469 = k

+ First order series reaction $A \rightarrow R \rightarrow S$

$$-r_A = k_1 C_A$$

$$-r_s = k_2 C_A$$

Since first order series reactions are irreversible reactions, k can be found in the same procedure for each reaction as in the first order irreversible reaction to be $k_1 = 0.2017$ and $k_2 = 0.134$.

- First order reversible reaction $A \leftrightarrow R$

$$-r_A = k_1 C_A - k_2 C_R$$

$$C_A = C_{Ao}(1 - X_A)$$

$$C_R = C_{Ao} X_A$$

$$\frac{dC_R}{dt} = \frac{-dC_A}{dt} = C_{Ao}\frac{dX_A}{dt} = k_1 C_{Ao}(1 - X_A) - k_2 C_{Ao} X_A$$

$$\frac{dX_A}{dt} = k_1(1 - X_A) - k_2 X_A$$

At equilubrium

$$\frac{dC_A}{dt} = 0, \frac{dX_A}{dt} = 0$$

$$K_c = \frac{C_{Re}}{C_{Ae}} = \frac{X_{Ae}}{(1 - X_{Ae})}$$

And the equilibrium constant to be

$$K_c = \frac{k_1}{k_2}$$

$$\frac{dX_A}{dt} = k_1(1 - X_A) - \frac{X_A}{K_c}) = k_1(1 - X_A - \frac{X_A(1 - X_{Ae})}{X_{Ae}}) = k_1\frac{X_{Ae} - X_A}{X_{Ae}}$$

$$\frac{dX_A}{dt} = k_1\frac{X_{Ae} - X_A}{X_{Ae}}$$

$$\int_0^X \frac{dX_A}{X_{Ae} - X_A} = \int_0^t \frac{k_1}{X_{Ae}} dt$$

$$-ln\left(\frac{X_{Ae} - X_A}{X_{Ae}}\right) = \frac{k_1 t}{X_{Ae}}, X_{Ae} = \frac{C_{Ao} - C_{Ae}}{C_{Ao}}$$

or

$$-ln\left(\frac{C_A - C_{Ae}}{C_{Ao} - C_{Ae}}\right) = \frac{k_1 C_{Ao} t}{C_{Ao} - C_{Ae}}$$

- Reaction order $n < 1$

$$r_A = -kC_A{}^n$$

$$\frac{-dC_A}{dt} = r_A = kC_A{}^n$$

After linearization

$$\ln\left(\frac{-dC_A}{dt}\right) = n\,\ln C_A + \ln k$$

Plot $\ln\left(\frac{-dC_A}{dt}\right)$ vs $\ln C_A$

Slope = 0.6542 = n

Intercept = $\ln k$ = - 0.5044

$k = e^{-0.5044} = 0.60387$

- Reaction order $n > 1$

The same procedure for reaction order $n < 1$ was applied to find that

Plot $\ln\left(\frac{-dC_A}{dt}\right)$ vs $\ln C_A$

Slope = 0.6615 = n

Intercept = $\ln k$ = - 1.5287

$k = e^{-1.5287} = 0.21682$

II. Data Sheet

Development of Kinetic Rate Equation
from Hydraulic Analog Methods Data Sheet

Time Volume (cm³)	1st — Reaction (Sec) Volume / Time	1st(w) — Reaction (mL) Volume / Time	Reaction Volume / Time	n<1 — Reaction (Sec) Volume / Time	n>1 — Reaction Volume / Time
50		5, 5.8	0	0	
49		10.6	5.8	18.3	
48		15.8	10.6	27	
47		8.3	15.8	35	
46		27.2	21.3	44	
45		23.6	27.2	53	
44		32.7 / 38.1	32.6	62	
43		38.1	38.1	72	
42			44	81	
41			50.3	89	
40			57.1	99.4	
39			62.9	109	
38			70.4	118.7	
37			76.6	128.7	
36			84.3	138.8	
35			91.9	148	
34			99.8	158.4	
33			108.2	168.1	
32			117.1	178.1	
31			126.3	189.4	
30			135.4	200	
29			145.8	221.5	
28			156.16	232	
27			168.17	242	
26			181.3	253.6	
25			194.5	265.6	
24			209	277	
23			227.4	289	
22			245.5		
21			268.8		
20			292		
19			324.1		
18			366.6		
17			420.8		
16			422.2		
15		15.5	425.2		
14					
13					
12					
11					
10					
9					
8					
7					

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Development of Kinetic Rate Equation
from Hydraulic Analog Methods Data Sheet

1st order irreversible run   1st order series run

Time (min) Volume	Reaction (s) Volume time	Reaction Volume	Reaction Volume	A Reaction Volume	B Reaction Volume
1 50 47	50 0			47.7	ø 48.5
2	11			61	44
3	10.			11.8	41.3
4	13			16.6	40
5	10			21	39.7
6	11			25	40
7	11			28.2	40.5
8	13			31.1	40.1
9	14			33.7	41.9
10	11			36	43.8
11	13			38	43.5
12	14			39.6	44
13	11			41.2	44.6
14	12			42.5	45.1
15	11			43.6	45.7
16	14			44.5	46
17	14			45.3	46.5
18	13			46	46.9
19	15			46.6	47
20	15			47.5	47.4
21	16			47.5	47.5
22	17			47.7	48 48
23	19				48
	15				48.3
	18				48.5
	20				
	21				
	20				
	20				
	23				
	25 23				
	25				
	28				
	29				
	31				
	28 30				
	35				
	35				
	38				
	41				
	44				
	47				
	64				
	75 90				
	96				

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